Theory of stability of self-sustaining DC discharges at inception with application to negative corona

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The inception of self-sustaining DC discharges is analyzed in terms of the bifurcation theory. The existence of a non-physical solution with negative ion and electron densities must be taken into account in order to identify the bifurcation type. The bifurcation is transcritical for positive and negative corona discharges and, in more general terms, it is expected to be transcritical for all discharge configurations except for the parallel-plate discharge, where the bifurcation is pitchfork. General trends of the bifurcation theory suggest that the corona discharges should be stable immediately after the inception. This conclusion is tested numerically for negative coronas in atmospheric-pressure air in coaxialcylinder geometry. Two independent approaches have been used: (1) study of linear stability against infinitesimal perturbations with the use of an eigenvalue solver and (2) following the time development of finite perturbations with the use of a time-dependent solver.

The numerical model of non-thermal discharges in high-pressure dry air, described in [1], is employed in this work. In brief, the model comprises equations of conservation and transport of charged species, written in the drift-diffusion approximation, and the Poisson equation. Charge-particle species accounted for in the model are the electrons, one representative species of positive ions, which are mostly O^{+2} , and three species of negative ions, O^{+2} , O^+ , O^{+3} . Transport and kinetic coefficients are assumed to depend on the local reduced electric field and the neutral-gas temperature. Photoionization is evaluated by means of the three-exponential model [2]; the gas temperature is set equal to 300K. Calculations reported in this work are performed for a one-dimensional negative corona discharge between concentric-cylinder electrodes in dry atmospheric-pressure air, which is an appropriate test case for the theory. The calculations refer to the inner electrode radius of 0.1mm, the outer electrode radius of 2mm, and the pressure of 1atm. Stationary, eigenvalue and time-dependent solvers of COMSOL Multiphysics® are used.

Fig. 1: Stability of negative corona discharge against small perturbations.

The stability of the negative corona discharge is illustrated by Fig. 1, and was calculated within the framework if the linear stability theory following a procedure similar to that employed in [3]. The black circle marked A in the figure designates the ignition of the self-sustaining discharge. Results from stability analysis, reveal that states in section AB are associated with increments which are real and negative. Therefore, this section is stable and perturbations decay monotonically with time. Section BC is associated with increments which have negative real part and nonzero imaginary part. This section is still stable, but perturbations decay with oscillations. Section CD is associated with positive real part and nonzero imaginary part. This section is unstable, and perturbations eventually lead to Trichel pulses.

The stability of the negative corona discharge against finite perturbations was also studied. As an example, results are shown in Fig. 2 for the states *a-d* shown by diamonds in Fig. 1.The procedure is the same as in $[1, 4]$: a small perturbation is introduced in applied voltage and the evolution of the discharge is followed by means of a time-dependent solver. If a stationary state is reached, the discharge is deemed stable. The procedure also allows one to determine whether the growth or decay of the perturbations is monotonic or oscillatory.

One can see from Fig. 2 that state *a* is stable and the perturbations decay

Fig. 2: Stability of negative corona discharge against finite perturbations.

monotonically. States *b* and *c* are also stable, but the decay of perturbations is oscillatory. State *d* is unstable, the perturbations grow accompanied by oscillations. For higher voltages, the perturbations lead to Trichel pulses. Detailed results will be given at the conference.

Numerical results are in perfect agreement with each other. In particular, the theory, the linear stability analysis, and the time-dependent modelling show that the negative corona is pulseless, immediately after the ignition. The stability is lost on harmonic perturbations, which evolve into Trichel pulses. Results are of theoretical interest and offer insights into physics of negative coronas. A number of interesting questions arise for future works, e.g., investigating the mechanism of the oscillations, exploring the potential effect of negative ions in this mechanism, and understanding the transition from harmonic oscillations to Trichel pulses and the potential effect of negative ions in this transition. A major challenge is to find the conditions under which a stable negative corona discharge occurs over a substantially wider range of voltages than 10V , in order to facilitate its unambiguous observation in experiment. This work has shown that results of linear stability analysis are in excellent agreement with results of time-dependent modelling of finite perturbations. Therefore, future numerical investigations of stability of negative corona discharges are likely to employ the latter approach, as was done in [4].

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