Macroscopic parameterization of positive streamers in air: velocity, radius, field etc.

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Streamer heads are essential building blocks of multi-streamer processes. They determine the growth of streamer trees or bursts. But velocity v and radius R of streamer heads can vary by orders of magnitude, and other macroscopic parameters vary as well, such as the background electric field $E_{\rm bg}$, the background electron density $n_{\rm bg}$, the maximal electric field $E_{\rm max}$, the charge content Q of the streamer head, the conductivity of the streamer channel $n_{\rm ch}$ and the degree of chemical activation of the medium.

We present a new axial model to approximate streamer heads:
We start from the fluid streamer model:

$$\begin{aligned}
\text{Electron density:} \quad & \partial_t n_e = \nabla\cdot(\mu \mathbf{E}\, n_e) + \alpha\, \mu E\, n_e + S_{ph}, \\
\text{Ion density:} \quad & \partial_t n_+ = \alpha\, \mu E\, n_e + S_{ph}, \\
\text{Poisson equation:} \quad & \nabla\cdot\mathbf{E} = \frac{\rho}{\epsilon_0}, \qquad \rho = e\,(n_+ - n_e).
\end{aligned}$$

In a coordinate system moving with the streamer velocity v, the operator ∂_t is replaced by $-v\partial_z$.
The electric field E consists of background field $E_{\rm bg}$ plus field enhancement generated by the charge layer. The layer is approximated as spherically symmetric with the unknown charge $q(r)$ where r is the radial coordinate:

$$E = E_{\rm bg} + \frac{q(r)}{4\pi\epsilon_0\, r^2}, \quad q(r) = \int_0^r 4\pi\, r'^2\, dr'\, \rho(r'), \quad q(r) = \begin{cases} 0 & \text{for } r=0 \\ Q & \text{for } r>R \end{cases}$$

With background ionization, this model can be rewritten as **three first order ordinary differential equations (ODEs)** for $n_e(z)$, $n_+(z)$ and $q(z)$ on the streamer axis, and easily solved.
When photo-ionization is included, some implicit equation needs to be solved iteratively.

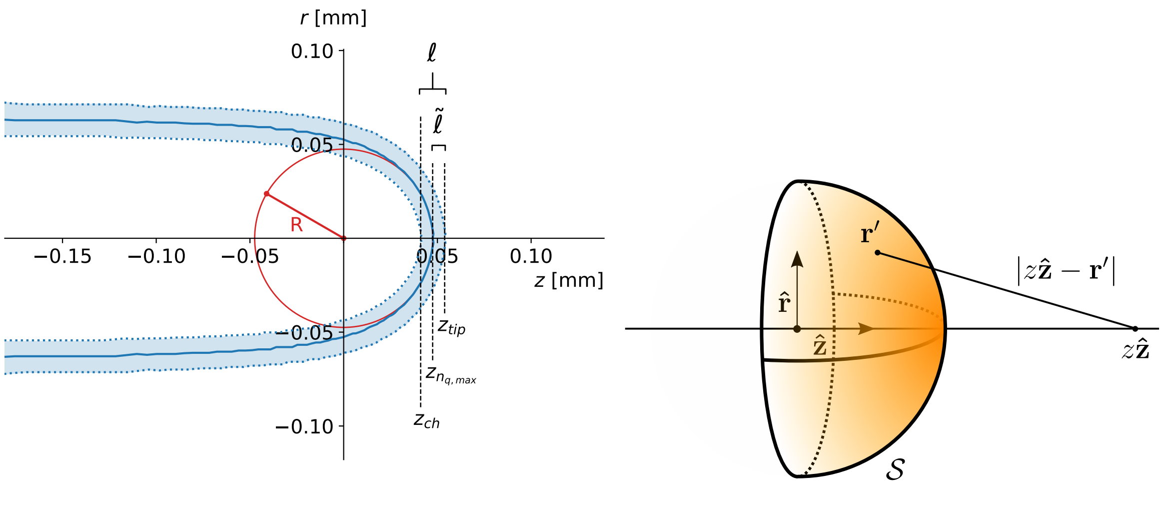

Fig. 1: **How to build an axial model for a streamer head:**
Left: Parameterization of a streamer head derived from a fluid streamer simulation in [1]. The maximal charge density as a function of z is indicated by a blue line. The red circle with radius R is fitted to this line.
Right: Calculation of photo-ionization on the streamer axis ahead of the charge layer.

The new ODE-model approximates the full 3D fluid model well, it largely relaxes computational demands, it relates macroscopic parameters to each other, and it allows for convenient parameter sweeps.

For example, we get relations between v, R, $E_{\max}$ and n_{ch} that depend on the transport and reaction coefficients of the gas in question. For air with photo-ionization, the relations are shown in Fig. 2.

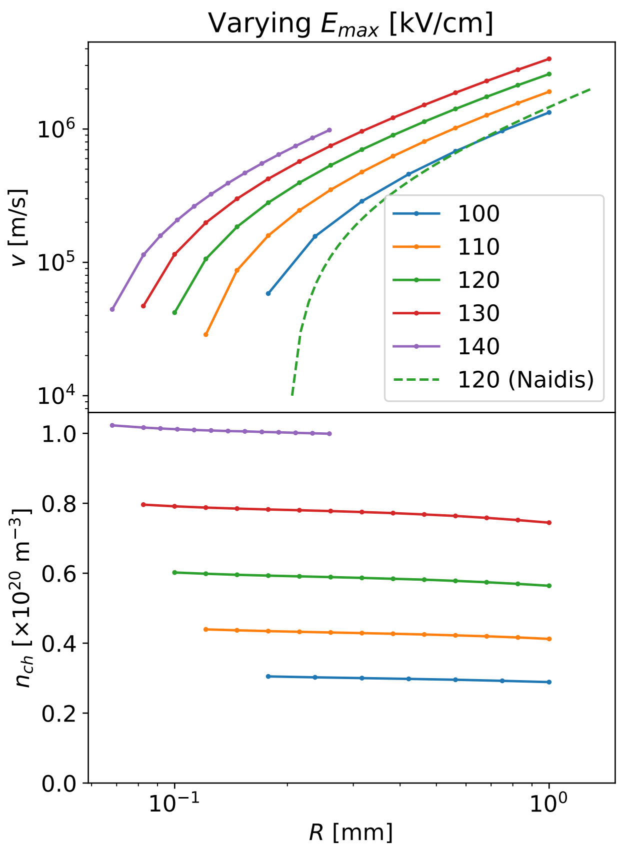

Fig. 2: **Relations between v, R, $E_{\max}$ and the generated channel conductivity n_{ch}** for steadily propagating streamer heads with photo-ionization in air under normal conditions. The colors indicate the values of the maximal electric field. The label [Naidis] indicates an earlier, less systematic approximation by Naidis [2].

How to read:
If radius R and velocity v of a streamer head are measured, the maximal electric field $E_{\max}$ can be read from the upper plot, and in a next step the channel conductivity n_{ch} can be read from the lower plot. It can be seen as well, that n_{ch} strongly depends on the maximal field $E_{\max}$ and little on the streamer radius R.

The results of ODE-approximation and macroscopic parameterization can be used
1. to construct computationally efficient multi-streamer models, and
2. to extract experimental data that are difficult to measure, as $E_{\max}$ in the example above.

[1] Dennis Bouwman, Hani Francisco and Ute Ebert, *Plasma Sources Sci. Technol.* **22** (2023) 075015.
[2] G.V. Naidis, *Phys. Rev. E* **79** (2009) 057401.