Solving steady and unsteady multi-physics problems using a hybridized discontinuous Galerkin solver

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Inductively coupled plasma (ICP) facilities possess unique properties such as accurately replicating the thermodynamic conditions of atmospheric reentry and enabling the examination of thermal shields for rockets and satellites. One of the most powerful ICP facility in the world is the plasmatron at von Karman Institute for Fluid Dynamics in Belgium, boasting a power output maxing out at 1200 kW and temperatures reaching 10000 K. The behaviour of reentry plasma in atmospheric conditions can be highly complex, featuring various strongly coupled physical phenomena. This complexity of the ICP facility needs to be predicted in the most accurate way to enhance the quality of the testing which led to the need of employing numerical methods. The development of the numerical method for ICP can be found for instance in [1] and [2]. ICP simulation codes are traditionally based upon the finite volumes method (FVM). The primary drawback of the FVM is that capturing high-temperature gradients near the wall region requires a high number of elements (cells) in that specific area. Another inherent limitation of the FVM is its low-order nature, making it challenging for studying the unsteadiness of the flow, including hydrodynamic instability or turbulence. In this contribution we will discuses an alternative approach providing high-order methods based upon the family of discontinuous Galerkin (DG) methods.

One of challenges in the description of ICP simulations is in managing multi-physics aspects involving calculation of flow, temperature and electric fields. In the language of mathematical physics, we simulate different physics over separate domains which are interconnected with specific interface conditions. One can approach multi-domain simulations using two distinct strategies. The first strategy is staggered, where individual solvers handle the domains, and these solvers only exchange data. The staggered approach can be either strongly or loosely coupled. In case of strongly coupled it is subiterated until the steady state solution is reached. In case of loosely coupled approach one forward step is performed successively for each of the domains. In both approaches, the information collected by one domain at its boundary is considered as a known quantity, although it is the result of the computations performed in another domain. However, these strategies may lead to instabilities, and there is a substantial body of literature on stability analysis of coupled methods (*e.g* [3]). Second possible strategy is a monolithic one. In this case the system is solved as a whole. The monolithic approach is allowing for faster and stable convergence to the solution. Additionally, this strategy has a profound impact on the system structure as it links the unknowns from distinct domains. In our work, a monolithic strategy is devoted for the hybridized discontinuous Galerkin (HDG) method [4], [5].

In this contribution we discusses two studies to demonstrate the capability of solving both steady and unsteady multi-physics problems using the HDG solver and further developing it for subsequent implementation in the ICP modelling. The first study solves steady conjugated heat transfer (CHT) problem, where the analytical solution is used as a benchmark for numerical solutions. In the CHT scenario, full problem is divided into the two subdomains. In the first subdomain, we solve heat transfer in the solid material, with governing equation being the heat diffusion equation (4). The second subdomain is representing fluid domain and is governed by Navier-Stokes equations (1) - (3).

Fluid

$$\frac{\partial u_x}{\partial x} = 0, \tag{1}$$

$$-\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u_x}{\partial y^2} = 0, \tag{2}$$

$$\mu \left(\frac{\partial u_x}{\partial y}\right)^2 + k_f \frac{\partial^2 T_f}{\partial y^2} = 0. \tag{3}$$

Solid

$$\frac{\partial^2 T_s}{\partial y^2} = 0. \tag{4}$$

where k_s is the thermal conductivity coefficient for the solid, k_f is the thermal conductivity coefficient for the fluid, μ is the dynamical viscosity, u_x is the x-component of the velocity, p is the pressure, T_f is the temperature of the fluid and finally T_s is the temperature of the solid. Multi-physics sketch can be seen in the Fig.1.

In the second case, an unsteady CHT problem is solved. Formulation of the governing equations and domains the assignment is the similar to the steady state case, while a time dependent boundary condition is used for the solid domain, incorporating the time integration through the diagonally implicit Runge-Kutta method (DIRK) [6]. These two test cases are fundamental building blocks for implementation Kelvin-Helmholtz hydrodynamic instabilities of ICP hot jets within HDG solver.

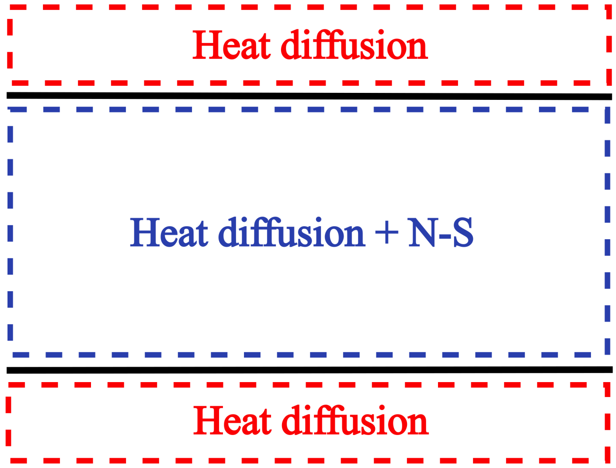

Fig. 1: Decomposition of domain in CHT problem. In the flow channel heat diffusion equation and the Navier-Stokes equations are solved. On the domains interface the only shared variable is the temperature, consequently only the heat diffusion equation is solved in the solid part.

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