

The electric potential in high power magnetron discharges: A toy model

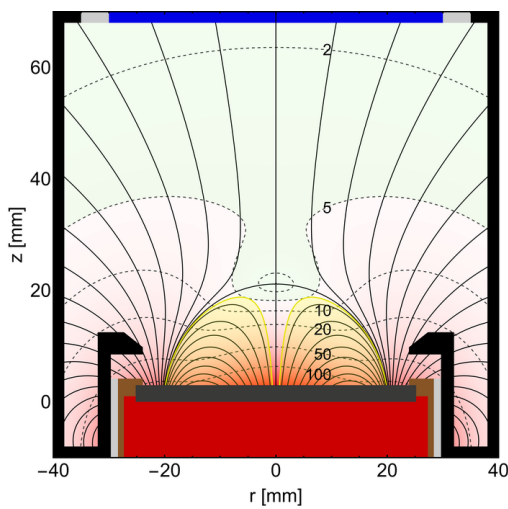
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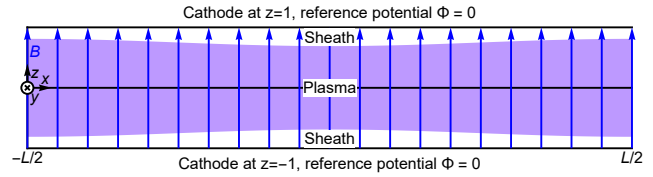
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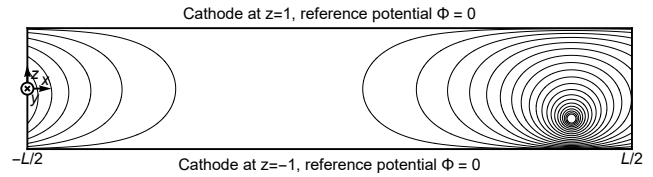
Partially magnetized plasmas, as employed in high power impulse magnetron sputtering (HiPIMS) and various other technological applications, often exhibit the spontaneous formation of “spokes”, dynamic ionization zones correlated with heightened plasma density and elevated electric potential. Conventional wisdom posits that accurately representing these phenomena requires moving beyond the assumption of quasineutrality, necessitating the inclusion of Poisson’s equation. This study addresses the issue using a physically and geometrically simplified setup, amenable to transparent solutions.



(a) Cross section of a circular planar magnetron and its magnetic field topology $\mathbf{B}(r, z)$ (to scale), mounted in a plasma chamber (not to scale).



(b) Simplified magnetron geometry with doubled cathode and constant magnetic field. The distribution of the plasma and the electric potential are invariant in y -direction.



(c) Two-dimensional Green’s function $G(x', z', x, z)$ which illustrates the reaction of the electric potential $\Phi(x, z)$ to a unit charge located at (x', z') .

Fig. 1: Magnetron modeling: From a realistic device to a simplified toy model.

Starting from a planar magnetron as shown in Fig. 1(a), we focus on the ionization zone where the magnetic field lines run from cathode to cathode. The geometry is simplified to be Cartesian, Fig. 1(b). The field lines are straightened and the field strength is set to be constant. The cathode is doubled. Invariance in y direction is assumed. The relation of the potential $\Phi(x, z)$ to the charge density $\rho(x, z)$ is given by Poisson’s equation. In dimensionless notation, it reads as follows, where the parameter ε denotes the ratio of the Debye length λ_D to the field line half length H :

$$-\varepsilon^2 \left(\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial z^2} \right) = n_i(z) - n_e(x, z), \quad (1)$$

$$\Phi(x, z) \Big|_{z=\pm 1} = 0.$$

The x -invariant ion density $n_i(z)$ is taken from an external model and considered fixed in this context. The electrons are bound to their field lines so that the integrated number density $N_e(x)$ is a given. However, they can freely move along the field lines and relax to Boltzmann equilibrium:

$$n_e(x, z) = \exp(\Phi(x, z) - V(x)) = N_e(x) \frac{\exp(\Phi(x, z))}{\int_{-1}^1 \exp(\Phi(x, z')) dz'} \quad (2)$$

If the number density per field line is constant, $N_e(x) = \bar{N}_e$, the solutions of the model assume a typical discharge structure with quasineutral plasma and electron-depleted sheaths of thickness s :

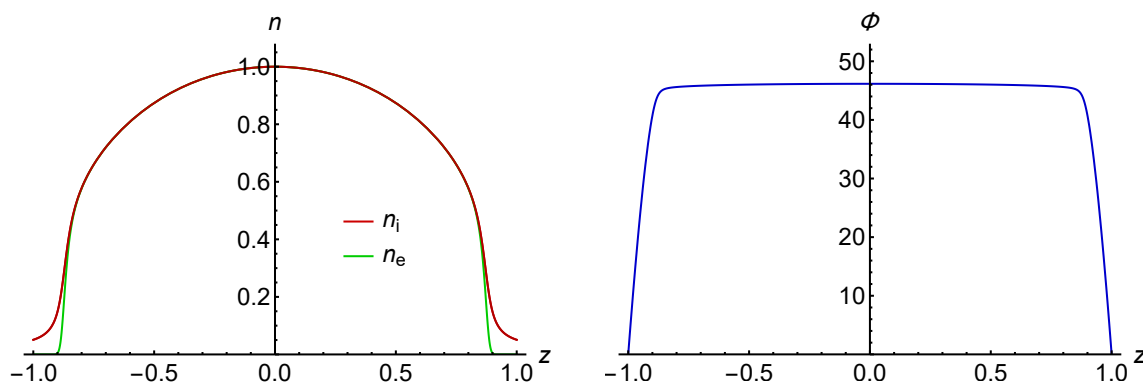
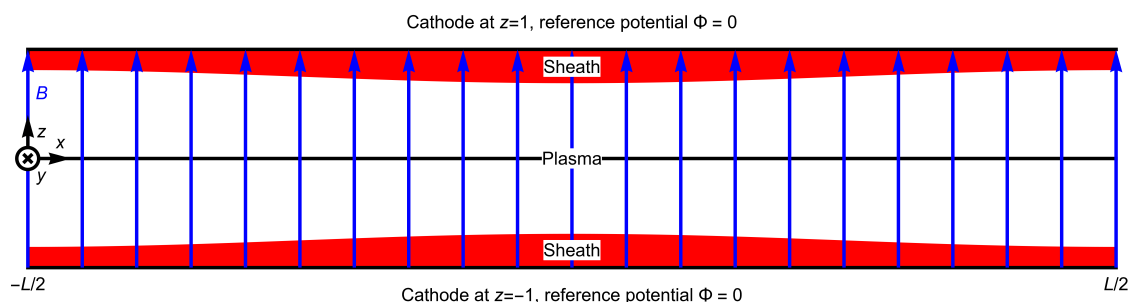
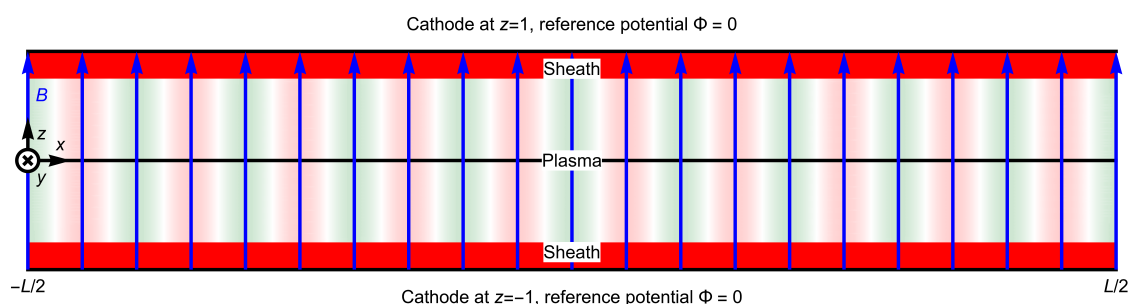


Fig. 2: Charge carrier densities (left) and potential (right) for a situation that is x -invariant.

The situation gets interesting when the number densities are perturbed, $N_e(x) = \bar{N}_e + \delta\tilde{N}_e \cos(kx)$. Our primary question: Will the extra charges accumulate in the sheath and keep the plasma quasineutral, or will they accumulate in the plasma and violate quasineutrality? It turns out that the answer depends on how the wavelength compares to the geometric mean of the field line length and the sheath thickness. Note that this scale is independent of the Debye length and thus not of a thermal nature.



(a) Long wavelength: Additional charges tend to accumulate in the boundary sheath.



(b) Short wavelength: Additional charges tend to accumulate in the plasma and violate quasineutrality.

Fig. 3: Response of the electron density $n_e(x, z)$ on perturbations $\delta N_e(x)$ of different wavenumbers k .

Thus, Poisson’s equation is indeed important for the modeling of high power magnetron discharges; the assumption of quasineutrality alone does not suffice. This fundamental insight extends beyond the simplified model explored in this study.

[1] K. Köhn, D. Krüger, D. Eremin, L. Xu, R.P. Brinkmann, *Plasma Sources Sci. Technol.* **32** (2023) 055012.
 [2] K. Köhn, D. Krüger, E. Kemaneci, L. Xu, D. Eremin, R.P. Brinkmann, *Plasma Sources Sci. Technol.* **30** (2021) 105014.